

UNCLASSIFIED

Defense Technical Information Center  
Compilation Part Notice

ADP014629

TITLE: Computer Simulation in Reliability

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: Proceedings of the Eighth Conference on the Design of  
Experiments in Army Research Development and Testing

To order the complete compilation report, use: ADA419759

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:

ADP014598 thru ADP014630

UNCLASSIFIED

# COMPUTER SIMULATIONS IN RELIABILITY

Jasper Dowling

U. S. Army Munitions Command, Picatinny Arsenal

ABSTRACT. The troublesome problems of calculating a realistic lower confidence limit for systems reliability from component results and writing algebraic probability expressions for complex systems have been investigated. Practical Monte Carlo procedures for routine use of high speed computers are described. Iterative procedures are explained which can:

- a. Save time in calculating the lower confidence limits of complex systems.
- b. Obtain point estimates of complex systems from Boolean expressions when the writing of algebraic equations is too difficult.

INTRODUCTION. This paper deals with two of the problem areas encountered in obtaining estimates of the reliability of a weapon system from component test data. The accuracy of the reliability estimate of a system is a function of the accuracies of the component reliability estimates. The calculation of the lower bound of the system estimate, or lower confidence limit, is one of the problem areas and is the first of the two discussed. The remainder of this paper explains a computer simulation technique for evaluating system Boolean expressions where algebraic probability expressions are not feasible.

## CONCLUSION.

1. It is concluded that the Monte Carlo technique is a valid and practical method of obtaining the lower confidence limit of a system. A 90% lower confidence limit of 0.88 obtained with the Monte Carlo method corresponds with that obtained by Garner and Vail (Reference 2) using a three component system in a series configuration with component reliabilities of 0.96, 0.97, and 0.99. Their 95% lower confidence limit of 0.88 using a different technique corresponded with the lower limit of 0.88 obtained with the Monte Carlo method.

2. A five component system of known system reliability was used as a standard in the second part of this report. The reliability of this known system was 0.98. By using the formula

$$\bar{p} \pm 2 \sqrt{\frac{pq}{n}}$$

it was determined with 95 % assurance that a Monte Carlo sample size of 78,400 would be needed to obtain values of  $0.98 \pm .0005$ . The system was run and resultant values of 0.9796 and 0.9799 were obtained. Although this is only an example it is concluded that the application of the simulation is valid, and hence the procedure is a practical, useful one.

### MAIN DISCUSSION .

1. To begin the discussion of the lower confidence limit let us assume a three component system in a series configuration. See Figure 1<sup>1</sup>. These components are assumed to be independent. This system reliability is  $0.96 \times 0.97 \times 0.99 = 0.92$  (to two decimal places). In this example each success ratio and hence, each binomial probability distribution is depicted for a sample size of 100. By using the binomial probability law we can compute a 90 % lower confidence limit for each of the components, as 0.92, 0.94, and 0.96 respectively. Their product  $0.92 \times 0.94 \times 0.96 = 0.83$ . This is not a 90 % confidence limit of the system.

2. To achieve the desired system distribution a Monte Carlo technique can be used to perform the product of the three component distributions. The probability of choosing a particular value from the component distribution will have to be equal to the distribution's probability (ordinate). To obtain this with a uniform random number generator, the three component binomial distributions are put into cumulative distributions. See Figure 2. A random number  $x$ ,  $0 \leq x \leq 1$  is generated. If this random number is less than the first value on the cumulative distribution (lowest ordinate) the value of the abscissa at this ordinate is assigned to the value of this component. If the random number is greater than the first ordinate, it is compared to the second ordinate. This continues until an ordinate is greater than the random number. The corresponding abscissa is assigned the value of the component. This is done for each of the three components and the three assigned values are substituted into the system equation and the equation solved. This is one point of the system distribution. As an example, assume the three random numbers .3321645, .21684290, and .93164200 are chosen. The corresponding reliabilities would be 0.95, 0.96, 1.0. The point on the system distribution would be the product of these three or 0.9120. This procedure is repeated many times to form the system distribution.

<sup>1</sup>All figures are contained in the appendix.

3. Figure 3 is the system distribution of Figure 1. This distribution is based on 5,000 points (repetitions) with a mean of 0.92, a standard deviation of 0.026 and a 90% lower confidence limit of 0.88. This lower confidence limit can be obtained by assuming normality and calculating the limit or by counting the lowest 500 points (10% of the total) of the system frequency distribution.

4. Figure 4 represents a 22 component system. Each component is represented by its binomial probability distribution based on a sample size of 100. It should be noted that although there are a number of similar components to be assigned the same probability, the algebraic equation must allow for 22 independent components. Assigning similar components the same Monte Carlo or simulated value will cause both higher and lower system values and hence an excessively high variance and a wider frequency distribution. For accuracy in counting the lowest cells for determining a counted confidence limit, the system frequency distribution is collected in small cell intervals and grouped after counting.

5. Figure 5 is the system distribution of Figure 4 based on 5,000 points. The mean value of this 22 component system is 0.98 and the lower 90% confidence limit is 0.97. It is interesting to point out that this distribution loses its symmetry as the sample size is decreased. The left hand tail becomes quite long and tapered. It should also be noted that although this distribution is essentially binomial, the sample size of the distribution becomes obscured in the formation of the distribution.

6. The second phase of this paper will be devoted to procedures for the evaluation of a system where the algebraic probability expression is not available. While dependency of components is a contributor in making the algebraic probability expression difficult, a system algebraic probability expression can become "not feasible" even when all components maintain their independence. Examples of situations that add to the complexity of a system probability expression are:

- a. Mechanical and electrical couplings
- b. One part of a system functions only if a second part of the system fails
- c. Multi-option channels.

7. Consider Figure 6. Clearly this is not a simple series-parallel diagram. There are 16 paths of success through this diagram,  $A_1 B_1 C_1 D_1 E_1 F_1$  etc. To write this as a system of 16 in parallel would require a considerable amount of algebra to account for repeating components in more than one of the 16 success paths. One approach to a solution to this problem is to redraw the diagram in a simple series-parallel configuration. See Figure 8. The trouble with this diagram is that there are twenty-four components represented where actually there are only nineteen physical components. Thus, the twenty-four components are not independent and in order to represent the system in a simple series-parallel configuration, it is necessary to draw the same component more than once. This procedure perhaps lessens the algebra required to write an algebraic probability expression, however, it would be preferable to consider a technique that doesn't require independence or an algebraic equation.

8. The basic notation and concept of Boolean algebra should be mentioned at this time. A plus sign is used to mean "or", and a dot is used to mean "and". The following expressions are thus introduced:

$$1 + 1 = 1$$

$$1 \cdot 1 = 1$$

$$1 + 0 = 1$$

$$1 \cdot 0 = 0$$

$$0 + 1 = 1$$

$$0 \cdot 1 = 0$$

$$0 + 0 = 0$$

$$0 \cdot 0 = 0$$

9. For purposes of illustration the example used earlier will be used again. Consider Figure 7. The procedure is as follows: Generate a random number (RN) from a uniform distribution:

If  $RN \leq$  reliability of component A, assign  $A = 1$

If  $RN \geq$  reliability of component A, assign  $A = 0$ .

Components B and C are treated similarly. Now, the probability expression in Boolean notation is  $P = A \cdot B \cdot C$  which reads  $P = A$  and  $B$  and  $C$ . In order for  $P$  to be a "1" all three components must be "1".

The probability of assigning a "1" to a component is the success ratio of the component. Hence, the probability of assigning all three components a "1" is the system's probability of success.

10. From Figure 8 the following expressions for different groups of components (X) can be obtained.

$$X_1 = A_1 \cdot B_1 + A_2 \cdot B_2$$

$$X_2 = C_1 \cdot (D_1 \cdot E_1 + D_3 \cdot E_2)$$

$$X_3 = C_2 \cdot (D_2 \cdot E_1 + D_4 \cdot E_2)$$

$$X_4 = X_1 \cdot (X_2 + X_3) \cdot F_1$$

$$X_5 = A_1 \cdot B_3 + A_2 \cdot B_4$$

$$X_6 = (C_3 + C_4) \cdot (F_2 + E_3 \cdot F_1)$$

$$\text{PROB} = X_4 + (X_5 \cdot X_6)$$

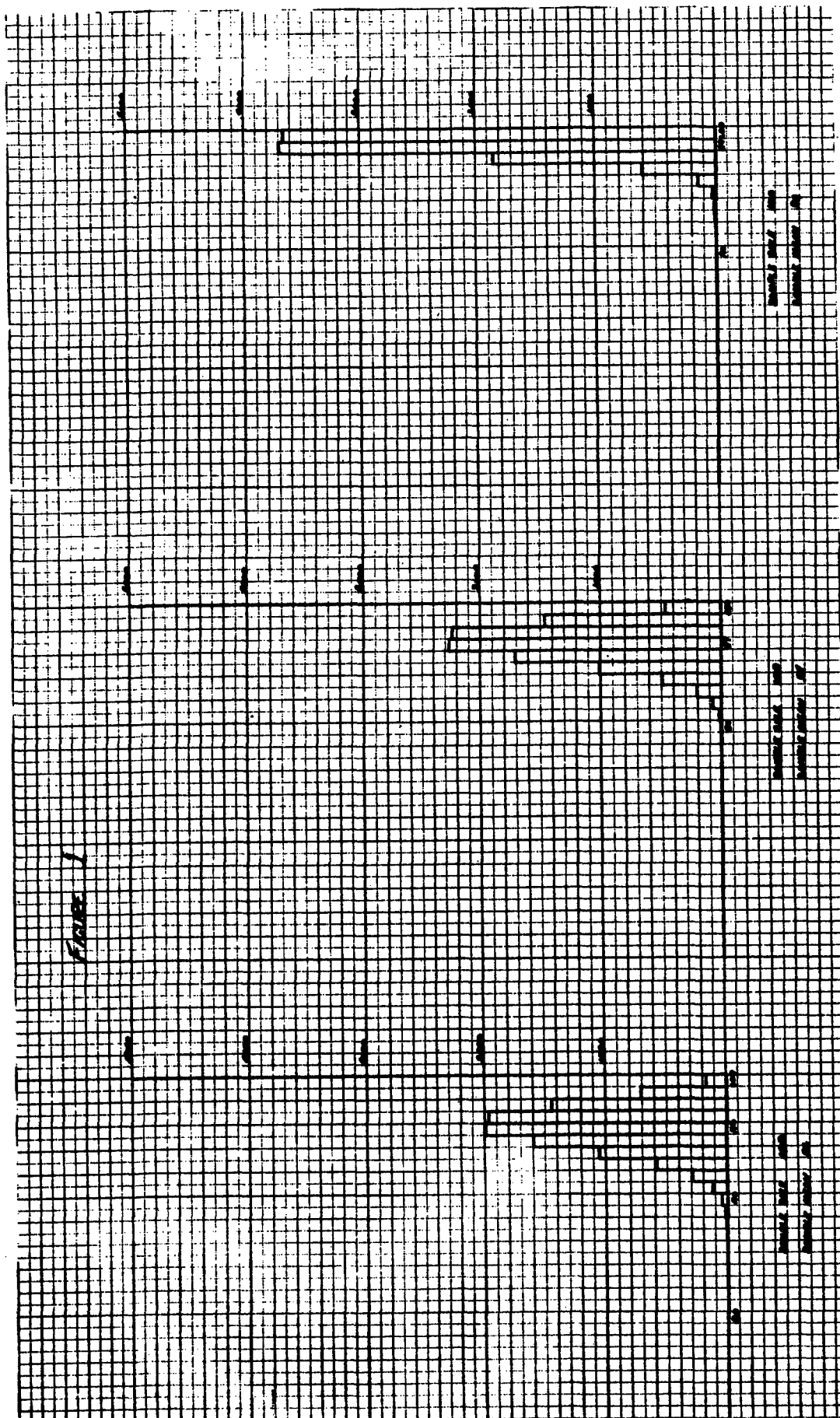
11. To solve this final expression a random number is generated and compared with component one,  $A_1$ .  $A_1$  is assigned a "0" or "1". This process continues until all nineteen (not twenty-four) components have been assigned a "0" or "1". IMPORTANT: when  $A_1$ ,  $A_2$ ,  $E_1$ ,  $E_2$ , and  $F_1$  are assigned a value they are assigned the same value everywhere they appear. Upon solving,  $X_1$  will be a "1" or "0". The probability of success of the system can be represented by

$$P = \frac{1}{n} \sum_{i=1}^n \text{PROB}_i \text{ where } n \text{ is the number of times the system is}$$

simulated.

References

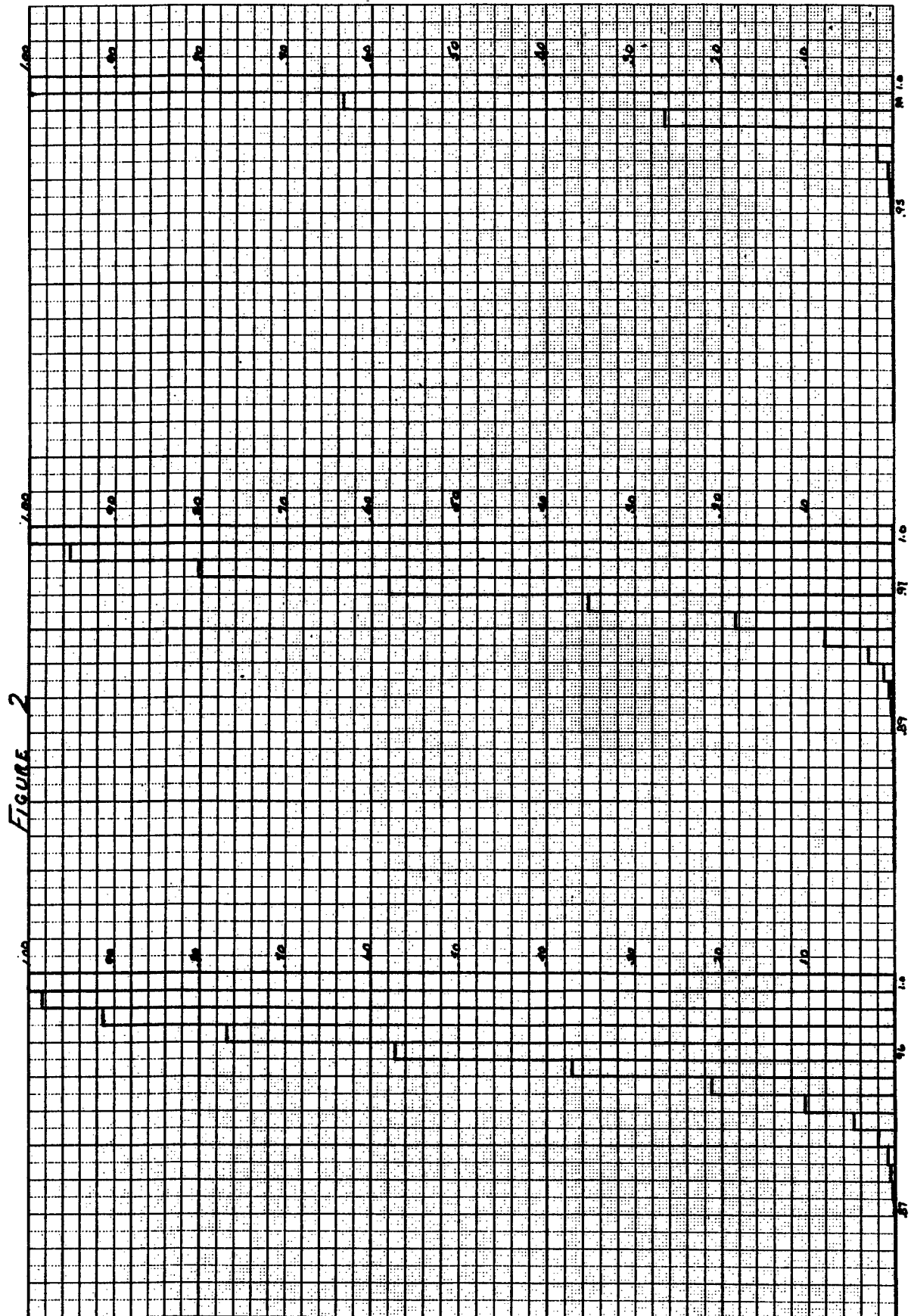
1. Orkand, Donald S. , "A Monte Carlo Method for Determining Lower Confidence Limits for System Reliability on the Basis of Sample Component Data", Picatinny Arsenal, Dover, New Jersey, June 1960.
2. Garner and Vail, "Confidence Limits for System Reliability", Systems Design, September-October 1961.





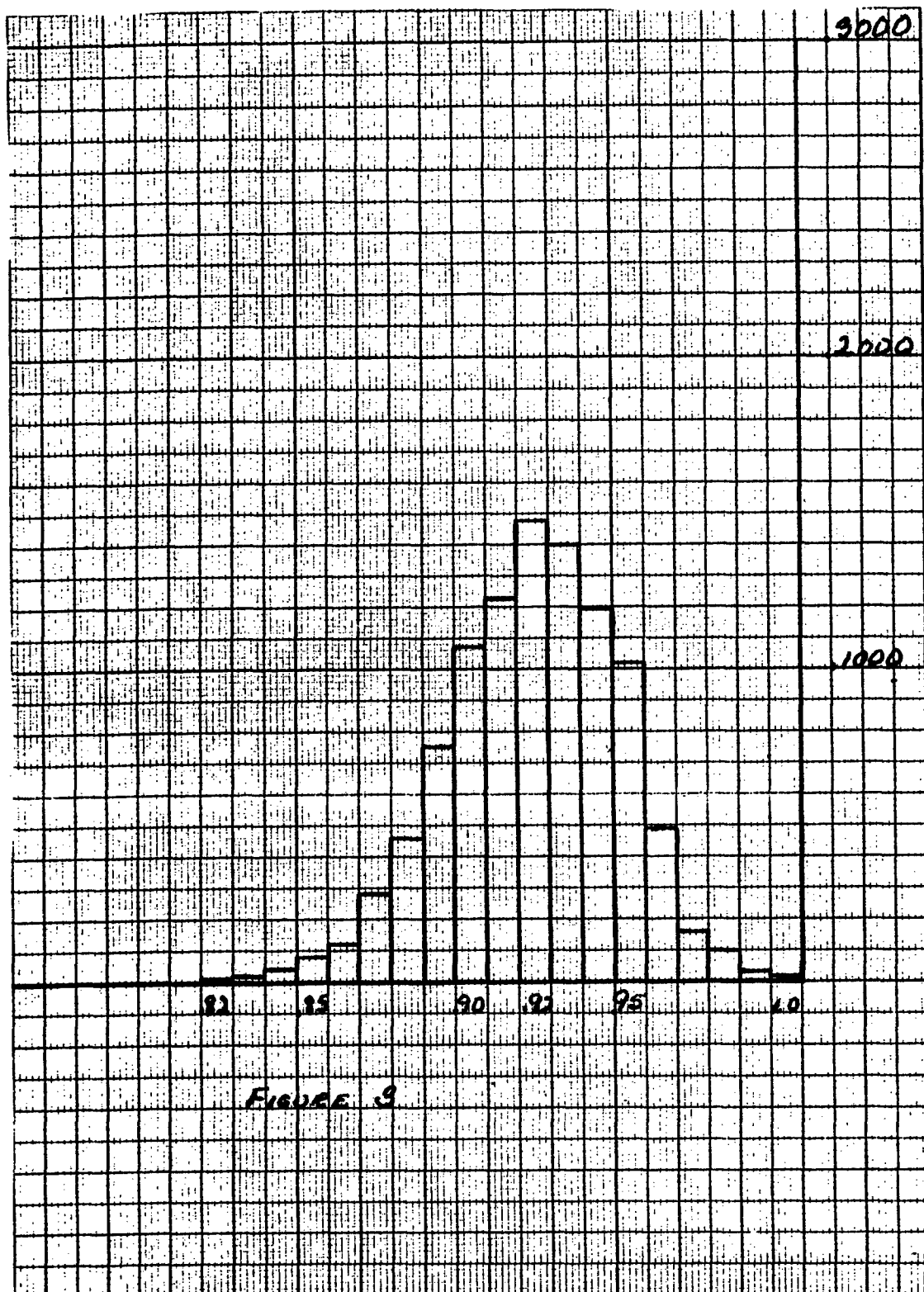
This Page Intentionally  
Left Blank

536



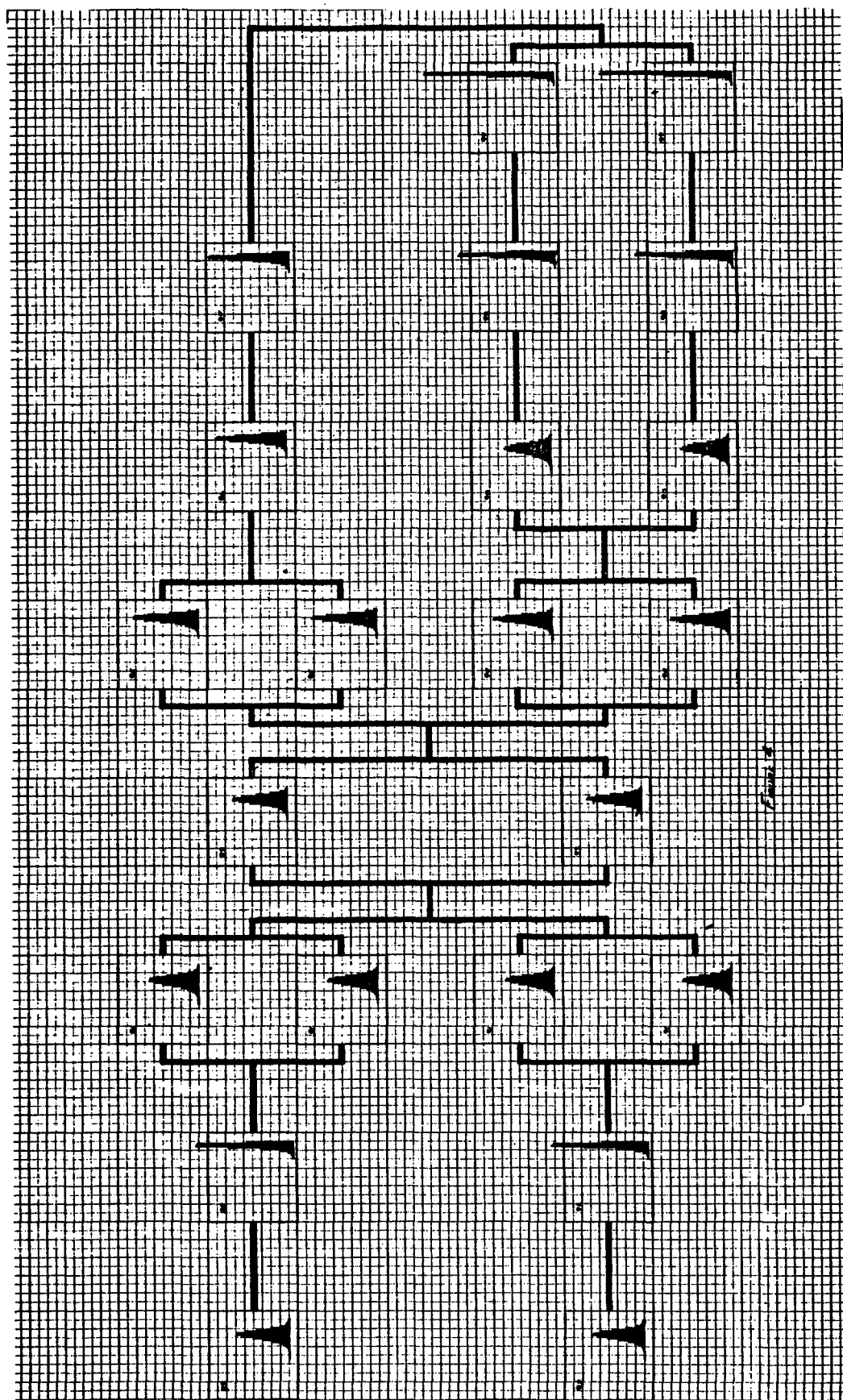
**This Page Intentionally  
Left Blank**

538



This Page Intentionally  
Left Blank

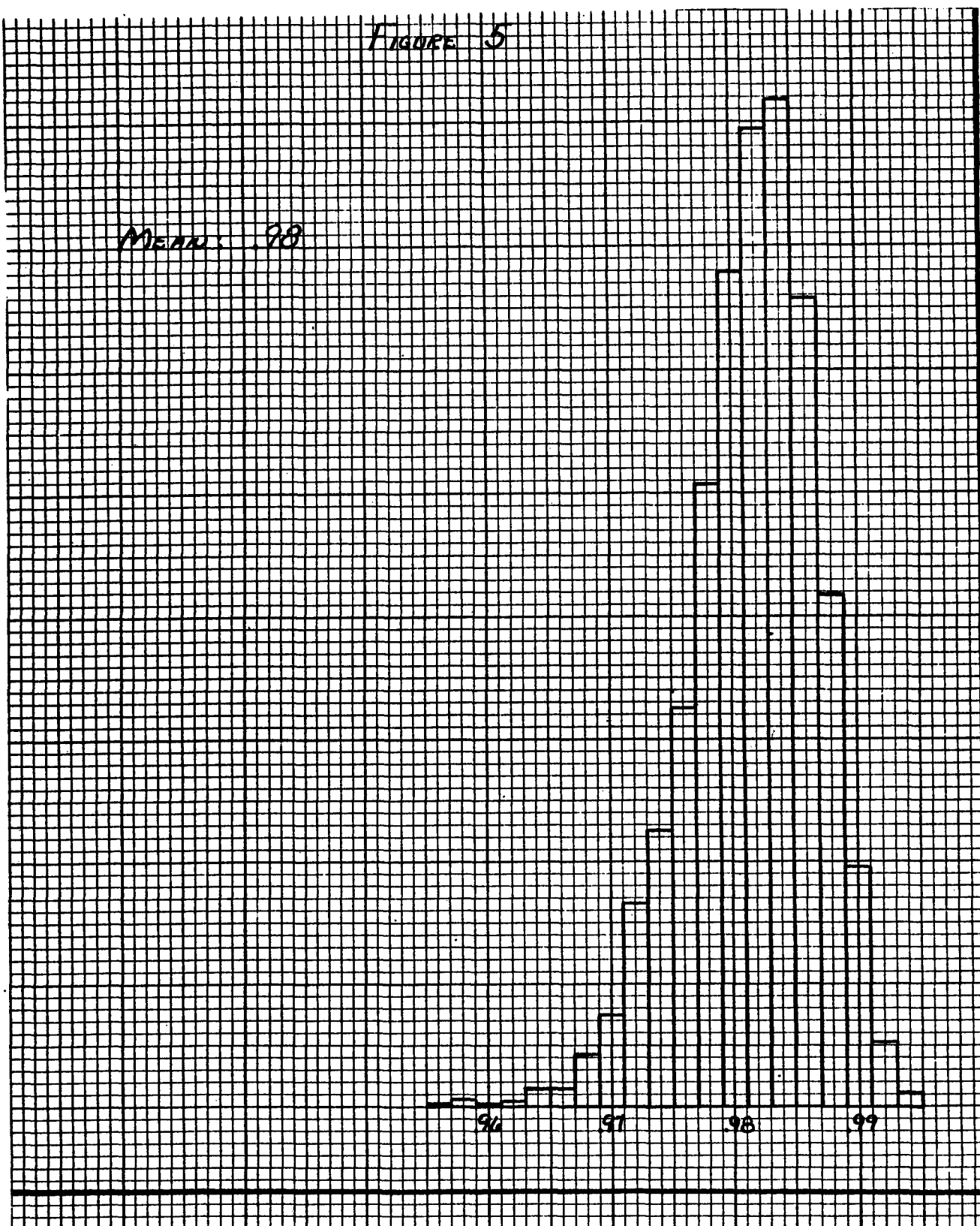
540



**This Page Intentionally  
Left Blank**

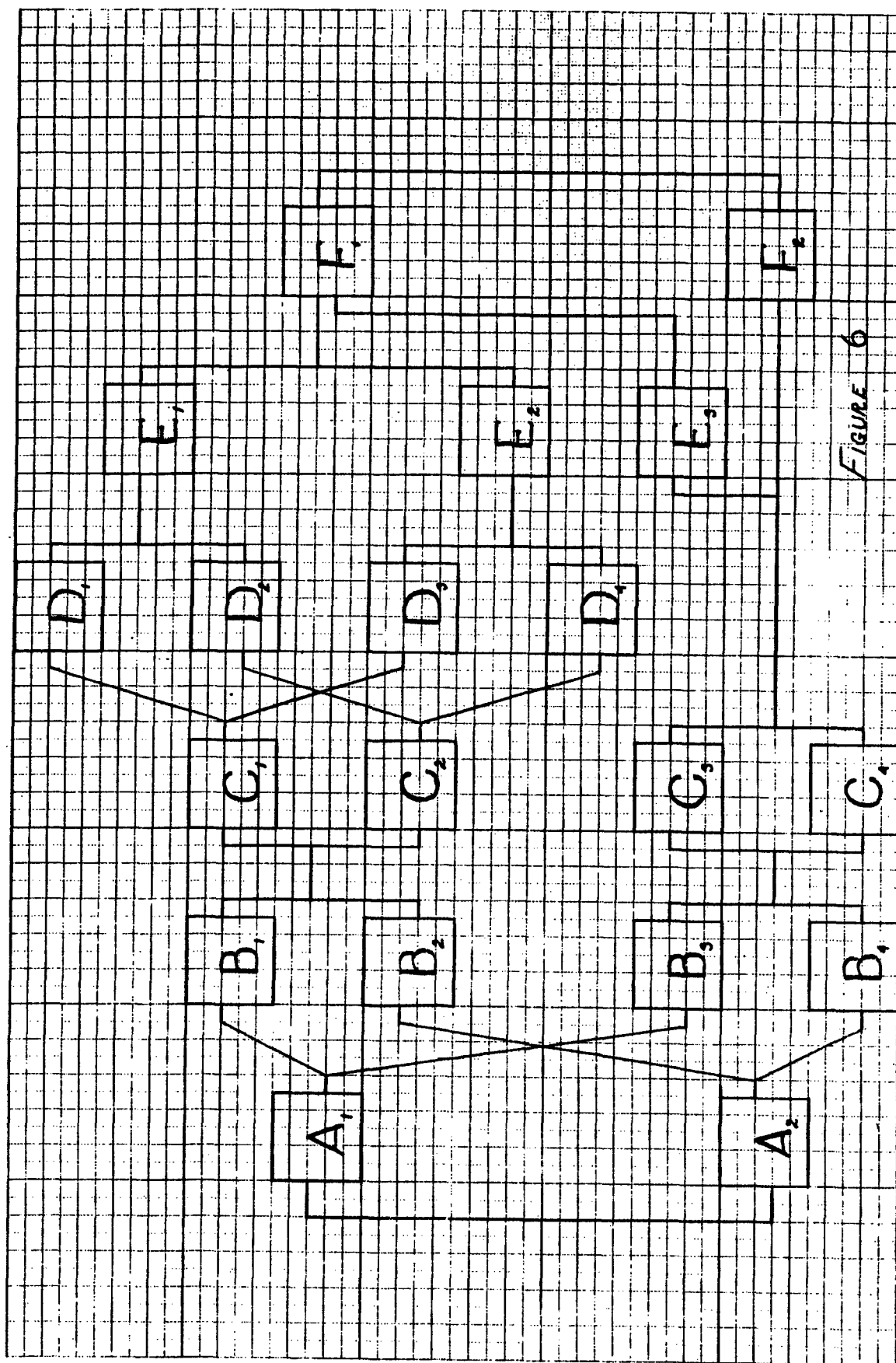
FIGURE 5

MEANS 98





**This Page Intentionally  
Left Blank**



**This Page Intentionally  
Left Blank**

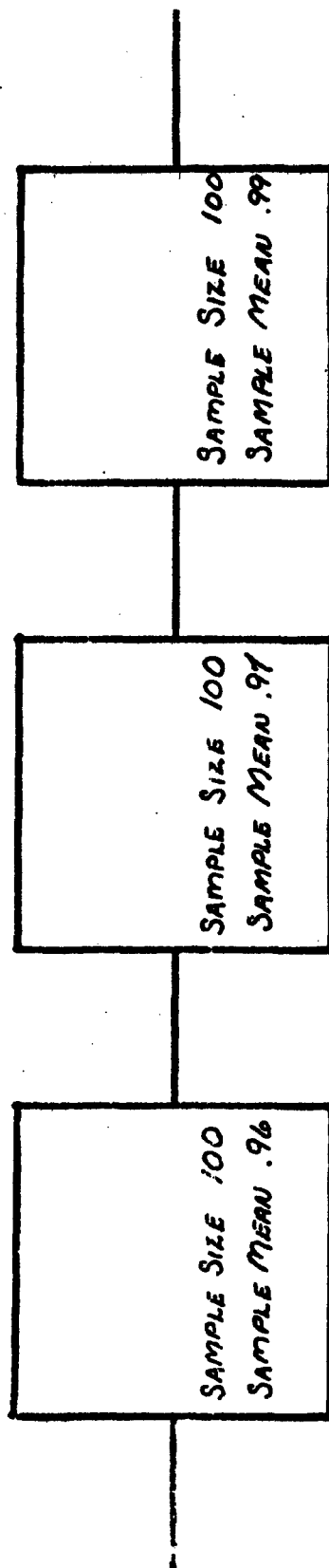
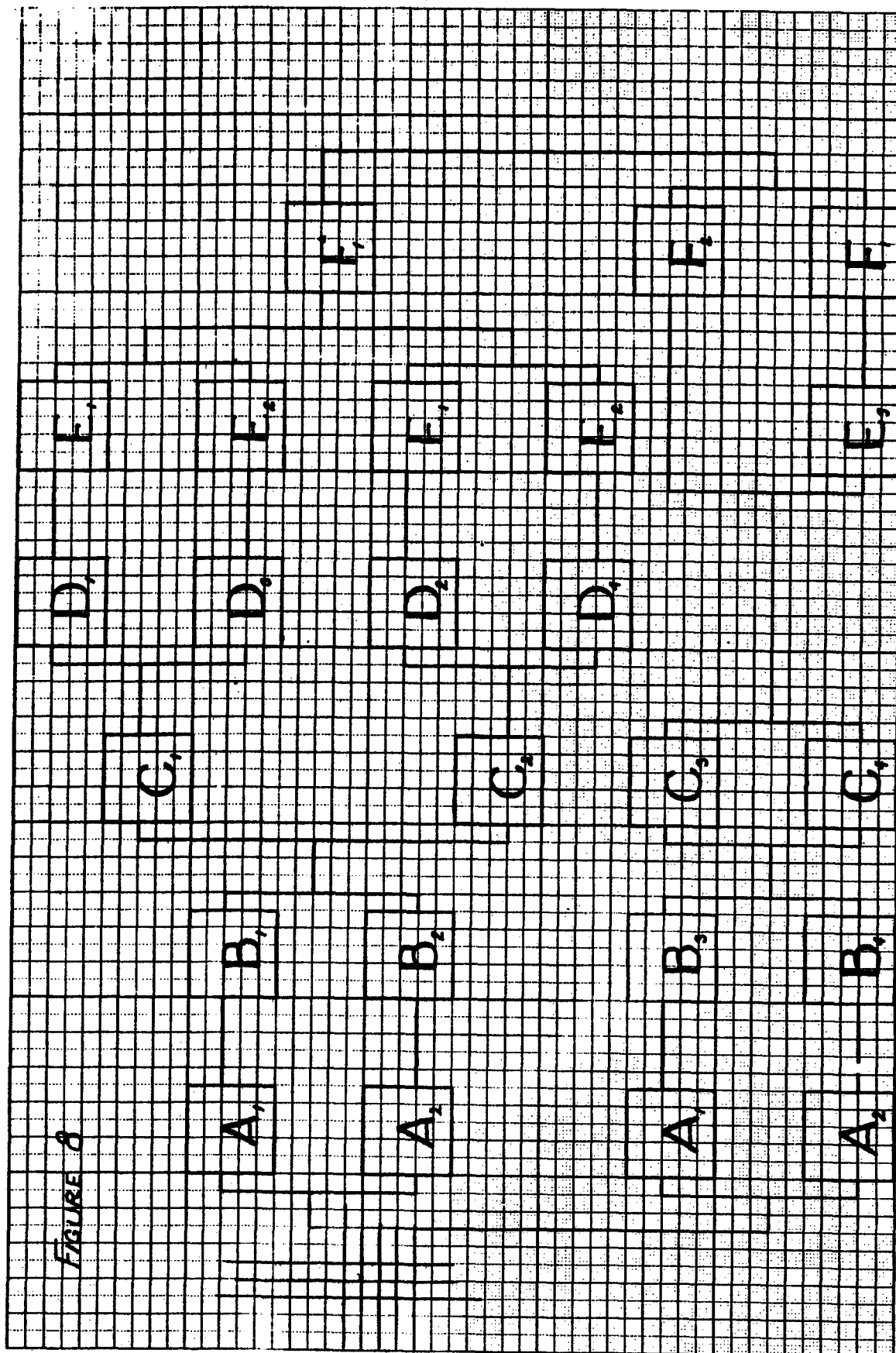


FIGURE 7

**This Page Intentionally  
Left Blank**



**This Page Intentionally  
Left Blank**